4 Vergelijkingen en ongelijkheden

Voorkennis Grafieken verschuiven

bladzijde 128

1 **a** $y = \frac{1}{4}x^2 + 3 \xrightarrow{\text{5 naar links}} y = \frac{1}{4}(x+5)^2 + 3$

b $y = -\frac{1}{3}x^2 - 4 \xrightarrow{\text{3 naar rechts}} y = -\frac{1}{3}(x-3)^2 - 4 \xrightarrow{\text{5 omhoog}} y = -\frac{1}{3}(x-3)^2 + 1$

2 **a** $y = -1\frac{1}{4}x^2 + 5 \xrightarrow{\text{4 omhoog}} y = -1\frac{1}{4}x^2 + 9$

b $y = -1\frac{1}{4}x^2 + 5 \xrightarrow{\text{3 naar rechts}} y = -1\frac{1}{4}(x-3)^2 + 5$

c $y = -1\frac{1}{4}x^2 + 5 \xrightarrow{\text{2 naar links}} y = -1\frac{1}{4}(x+2)^2 + 5 \xrightarrow{\text{3 omlaag}} y = -1\frac{1}{4}(x+2)^2 + 2$

d $y = -1\frac{1}{4}x^2 + 5 \xrightarrow{\text{1 naar rechts}} y = -1\frac{1}{4}(x-1)^2 + 5 \xrightarrow{\text{5 omlaag}} y = -1\frac{1}{4}(x-1)^2$

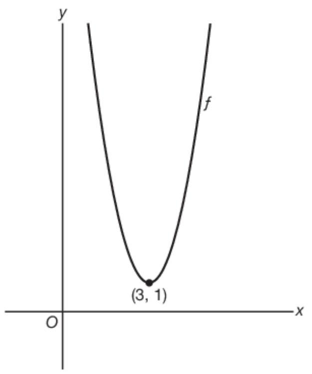
bladzijde 129

3 **a** $y = 2x^2 \xrightarrow{\text{3 naar rechts}} y = 2(x-3)^2 \xrightarrow{\text{4 omhoog}} y = 2(x-3)^2 + 4$
Dus 3 naar rechts en 4 omhoog.

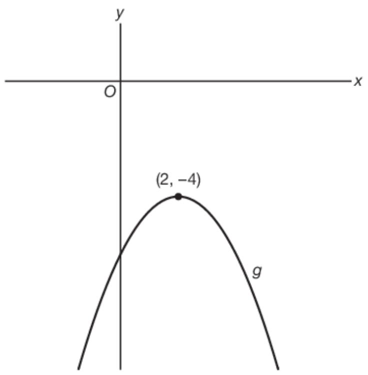
b $y = -x^2 + 1 \xrightarrow{\text{2 naar links}} y = -(x+2)^2 + 1 \xrightarrow{\text{1 omlaag}} y = -(x+2)^2$
Dus 2 naar links en 1 omlaag.

c $y = 1\frac{1}{2}(x-4)^2 + 5 \xrightarrow{\text{5 naar links}} y = 1\frac{1}{2}(x+1)^2 + 5 \xrightarrow{\text{8 omlaag}} y = 1\frac{1}{2}(x+1)^2 - 3$
Dus 5 naar links en 8 omlaag.

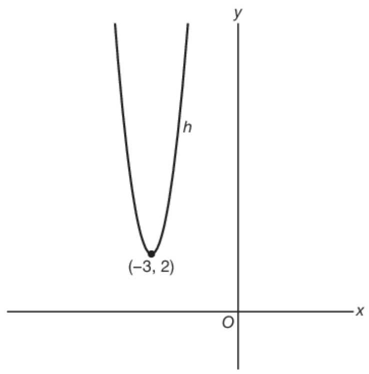
4 **a** De top is (3, 1).
$a = 2$, dus $a > 0$ en dit geeft een dalparabool.



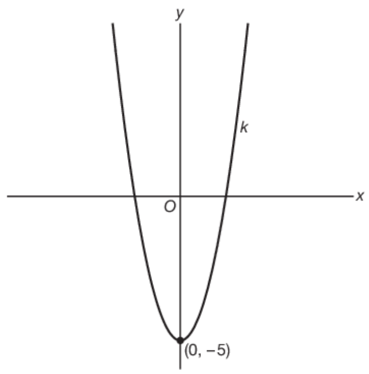
b De top is (2, −4).
$a = -\frac{1}{2}$, dus $a < 0$ en dit geeft een bergparabool.



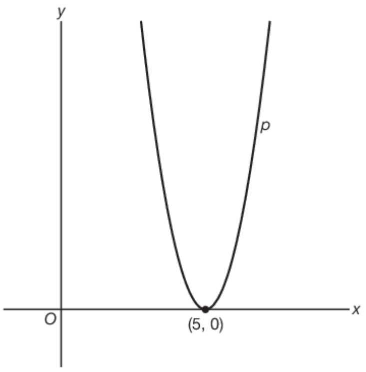
c De top is (−3, 2).
$a = 5$, dus $a > 0$ en dit geeft een dalparabool.



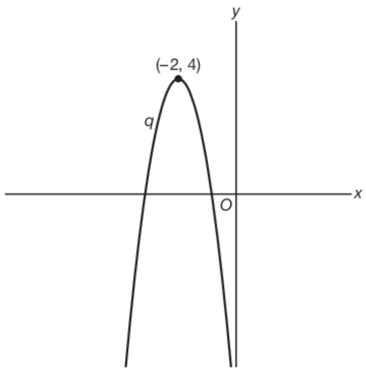
d De top is (0, −5).
$a = 2$, dus $a > 0$ en dit geeft een dalparabool.



e De top is (5, 0).
$a = 2$, dus $a > 0$ en dit geeft een dalparabool.



f De top is (−2, 4).
$a = -3$, dus $a < 0$ en dit geeft een bergparabool.



4.1 Stelsels vergelijkingen

bladzijde 130

1 **a** Tien broden kosten 16 euro.
Blijft over $60 - 16 = 44$ euro.
Dus kan hij nog $\frac{44}{0{,}40} = 110$ bolletjes kopen.
b 90 bolletjes kosten $90 \times 0{,}4 = 36$ euro.
Blijft dus over $60 - 36 = 24$ euro.
Dus kan hij nog $\frac{24}{1{,}60} = 15$ broden kopen.
c $1{,}6x + 0{,}4y = 60$

bladzijde 132

2 **a** $15x + 12y = 2520$
$12y = -15x + 2520$
$y = -1\frac{1}{4}x + 210$
b $3p - 2q = 16\frac{1}{2}$
$3p = 2q + 16\frac{1}{2}$
$p = \frac{2}{3}q + 5\frac{1}{2}$
c $5a - 2b = 16$
$-2b = -5a + 16$
$b = 2\frac{1}{2}a - 8$

3 **a** l: $3x - y = 6$

x	0	2
y	−6	0

m: $x + y = 1$

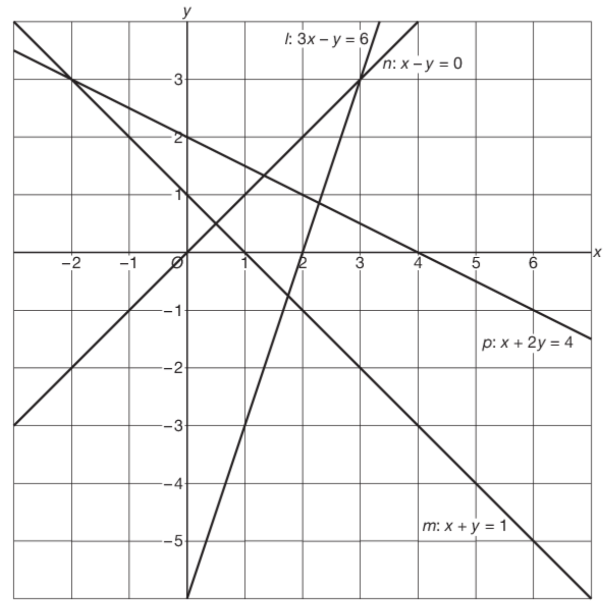
x	0	1
y	1	0

n: $x - y = 0$

x	0	1
y	0	1

p: $x + 2y = 4$

x	0	4
y	2	0



b l: $3x - y = 6$
$-y = -3x + 6$
$y = 3x - 6$
$rc_l = 3$

m: $x + y = 1$
$y = -x + 1$
$rc_m = -1$

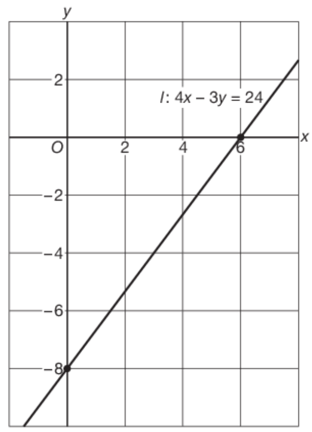
n: $x - y = 0$
$-y = -x$
$y = x$
$rc_n = 1$

p: $x + 2y = 4$
$2y = -x + 4$
$y = -\frac{1}{2}x + 2$
$rc_p = -\frac{1}{2}$

4 **a** l: $4x - 3y = 24$

Snijden met de x-as, dus
$y = 0$
$4x = 24$
$x = 6$
dus (6, 0).

Snijden met de y-as, dus
$x = 0$
$-3y = 24$
$y = -8$
dus (0, –8).



b $A(8, 3)$ invullen geeft $4 \cdot 8 - 3 \cdot 3 = 24$
$32 - 9 = 24$
Klopt niet, dus A ligt niet op l.

$B(18, 16)$ invullen geeft $4 \cdot 18 - 3 \cdot 16 = 24$
$72 - 48 = 24$
Klopt, dus B ligt op l.

$C(-30, -48)$ invullen geeft $4 \cdot -30 - 3 \cdot -48 = 24$
$-120 + 144 = 24$
Klopt, dus C ligt op l.

c $(16, p)$ invullen geeft $4 \cdot 16 - 3 \cdot p = 24$
$64 - 3p = 24$
$-3p = -40$
$p = 13\frac{1}{3}$

d $(q, 48)$ invullen geeft $4 \cdot q - 3 \cdot 48 = 24$
$4q - 144 = 24$
$4q = 168$
$q = 42$

5 **a** l: $3x - 4y = 7$
$-4y = -3x + 7$
$y = \frac{3}{4}x - \frac{7}{4}$
$rc_l = \frac{3}{4}$

m: $3x - 4y = -8$
$-4y = -3x - 8$
$y = \frac{3}{4}x + 2$
$rc_m = \frac{3}{4}$

b Omdat ze dezelfde richtingscoëfficiënt hebben, namelijk $\frac{3}{4}$.

c $A(5, 1)$ invullen in $3x - 4y = c$ geeft $3 \cdot 5 - 4 \cdot 1 = c$
$15 - 4 = c$
$c = 11$

d $B(3, -1)$ invullen in $3x - 4y = c$ geeft $3 \cdot 3 - 4 \cdot -1 = c$
$9 + 4 = c$
$c = 13$

Dus k: $3x - 4y = 13$.

6 $A(5, 8)$ invullen in $2x + y = c$ geeft $2 \cdot 5 + 8 = c$
$10 + 8 = c$
$c = 18$

Dus m: $2x + y = 18$.

7 $12x + 4y = 242{,}40$

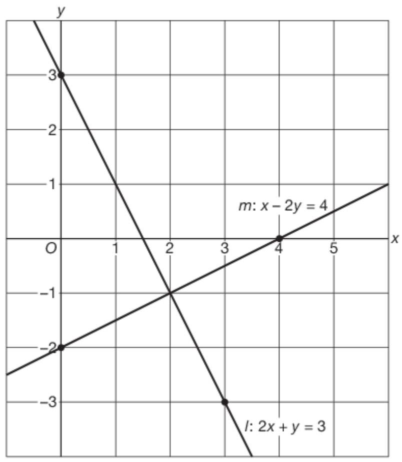
8 Stel x kaartjes van 10 euro en y kaartjes van 15 euro. Nu geldt $10x + 15y = 4300$.

9 **a** $l: 2x + y = 3$

x	0	3
y	3	–3

$m: x - 2y = 4$

x	0	4
y	–2	0



b (2, –1) is het snijpunt.
c (2, –1) is oplossing van $x - 2y = 4$ en van $2x + y = 3$.

bladzijde 134

10 **a**
$$\begin{cases} 5x - 4y = -8 \\ -x + 4y = -12 \end{cases} \quad +$$
$$4x = -20$$
$$x = -5$$
$$-x + 4y = -12$$
$$5 + 4y = -12$$
$$4y = -17$$
$$y = -4\tfrac{1}{4}$$

De oplossing is $(-5, -4\tfrac{1}{4})$.

b
$$\begin{cases} -2x + y = 7 \\ -2x + 3y = -1 \end{cases} \quad -$$
$$-2y = 8$$
$$y = -4$$
$$-2x + y = 7$$
$$-2x - 4 = 7$$
$$-2x = 11$$
$$x = -5\tfrac{1}{2}$$

De oplossing is $(-5\tfrac{1}{2}, -4)$.

c
$$\begin{cases} -x - 3y = -8 \\ -2x + 3y = -1 \end{cases} \quad +$$
$$-3x = -9$$
$$x = 3$$
$$-2x + 3y = -1$$
$$-6 + 3y = -1$$
$$3y = 5$$
$$y = \tfrac{5}{3} = 1\tfrac{2}{3}$$

De oplossing is $(3, 1\tfrac{2}{3})$.

11 **a** $\begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \end{cases}$ +

$5x - y = 23$

Nee, er is na het optellen geen variabele geëlimineerd.

b $\begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \end{cases}$ −

$x - 7y = -9$

Nee, er is na het aftrekken geen variabele geëlimineerd.

bladzijde 136

12 **a** $\begin{cases} 3x + 5y = -7 \\ 2x + y = 0 \end{cases} \begin{vmatrix} 1 \\ 5 \end{vmatrix}$ geeft $\begin{cases} 3x + 5y = -7 \\ 10x + 5y = 0 \end{cases}$ −

$-7x = -7$

$\left.\begin{array}{r} x = 1 \\ 2x + y = 0 \end{array}\right\} \begin{array}{l} 2 + y = 0 \\ y = -2 \end{array}$

De oplossing is (1, –2).

b $\begin{cases} 2x - 4y = 6 \\ 3x - y = 19 \end{cases} \begin{vmatrix} 1 \\ 4 \end{vmatrix}$ geeft $\begin{cases} 2x - 4y = 6 \\ 12x - 4y = 76 \end{cases}$ −

$-10x = -70$

$\left.\begin{array}{r} x = 7 \\ 2x - 4y = 6 \end{array}\right\} \begin{array}{l} 14 - 4y = 6 \\ -4y = -8 \\ y = 2 \end{array}$

De oplossing is (7, 2).

c $\begin{cases} 4x + y = 13 \\ x - 2y = 1 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 8x + 2y = 26 \\ x - 2y = 1 \end{cases}$ +

$9x = 27$

$\left.\begin{array}{r} x = 3 \\ 4x + y = 13 \end{array}\right\} \begin{array}{l} 12 + y = 13 \\ y = 1 \end{array}$

De oplossing is (3, 1).

13 **a** $\begin{cases} 5x + 2y = 69 \\ x + 3y = -7 \end{cases} \begin{vmatrix} 1 \\ 5 \end{vmatrix}$ geeft $\begin{cases} 5x + 2y = 69 \\ 5x + 15y = -35 \end{cases}$ −

$-13y = 104$

$\left.\begin{array}{r} y = -8 \\ x + 3y = -7 \end{array}\right\} \begin{array}{l} x - 24 = -7 \\ x = 17 \end{array}$

De oplossing is (17, –8).

b $\begin{cases} 2x - 5y = -19 \\ 5x + 4y = 35 \end{cases} \begin{vmatrix} 4 \\ 5 \end{vmatrix}$ geeft $\begin{cases} 8x - 20y = -76 \\ 25x + 20y = 175 \end{cases}$ +

$33x = 99$

$\left.\begin{array}{r} x = 3 \\ 2x - 5y = -19 \end{array}\right\} \begin{array}{l} 6 - 5y = -19 \\ -5y = -25 \\ y = 5 \end{array}$

De oplossing is (3, 5).

c $\begin{cases} 0{,}8x + 0{,}2y = 1 \\ 0{,}3x - 0{,}3y = 1{,}5 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 2{,}4x + 0{,}6y = 3 \\ 0{,}6x - 0{,}6y = 3 \end{cases}$ +

$3x = 6$

$\left.\begin{array}{r} x = 2 \\ 0{,}8x + 0{,}2y = 1 \end{array}\right\} \begin{array}{l} 1{,}6 + 0{,}2y = 1 \\ 0{,}2y = -0{,}6 \\ y = -3 \end{array}$

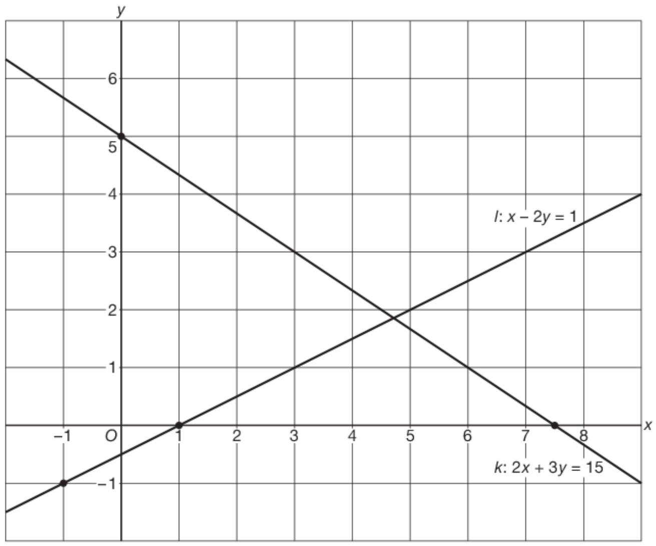
De oplossing is (2, –3).

14 **a** $k: 2x + 3y = 15$

x	0	$7\frac{1}{2}$
y	5	0

$l: x - 2y = 1$

x	-1	1
y	-1	0



b Je moet het stelsel $\begin{cases} 2x + 3y = 15 \\ x - 2y = 1 \end{cases}$ oplossen.

$\begin{cases} 2x + 3y = 15 \\ x - 2y = 1 \end{cases} \begin{vmatrix} 2 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 4x + 6y = 30 \\ 3x - 6y = 3 \end{cases}$ +

$7x = 33$

$x = \frac{33}{7} = 4\frac{5}{7}$, $2x + 3y = 15$ → $9\frac{3}{7} + 3y = 15$

$3y = 5\frac{4}{7}$

$y = 1\frac{6}{7}$

Het snijpunt is $(4\frac{5}{7}, 1\frac{6}{7})$.

15 **a** $\begin{cases} 3x - 2y = -12 \\ x + 4y = 38 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 6x - 4y = -24 \\ x + 4y = 38 \end{cases}$ +

$7x = 14$

$x = 2$, $x + 4y = 38$ → $2 + 4y = 38$

$4y = 36$

$y = 9$

Het snijpunt is (2, 9).

b $\begin{cases} 2x + 5y = 26 \\ 3x - 2y = 1 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix}$ geeft $\begin{cases} 6x + 15y = 78 \\ 6x - 4y = 2 \end{cases}$ −

$19y = 76$

$y = 4$, $2x + 5y = 26$ → $2x + 20 = 26$

$2x = 6$

$x = 3$

Het snijpunt is (3, 4).

16 **a** 'Leen heeft 50 munten' geeft $x + y = 50$.
'Het bedrag is € 87' geeft $x + 2y = 87$.

b $\begin{cases} x + y = 50 \\ x + 2y = 87 \end{cases}$ −

$-y = -37$

$\left.\begin{array}{r} y = 37 \\ x + y = 50 \end{array}\right\}$ $x + 37 = 50$
$x = 13$

Leen heeft 13 munten van 1 euro.

17 Stel het aantal kg appels dat de groenteman verkoopt x en het aantal kg peren y.
Uit de gegevens volgt het stelsel

$\begin{cases} x + y = 295 \\ 1{,}4x + 1{,}7y = 452 \end{cases} \begin{vmatrix} 1{,}4 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 1{,}4x + 1{,}4y = 413 \\ 1{,}4x + 1{,}7y = 452 \end{cases}$ −

$-0{,}3y = -39$

$\left.\begin{array}{r} y = 130 \\ x + y = 295 \end{array}\right\}$ $x + 130 = 295$
$x = 165$

De groenteman heeft die dag 165 kg appels verkocht.

18 Het totale aantal jaren is $15 \cdot 16{,}4 = 246$.
Stel het aantal jongens x en het aantal meisjes y.
Uit de gegevens volgt het stelsel

$\begin{cases} x + y = 15 \\ 15{,}6x + 16{,}8y = 246 \end{cases} \begin{vmatrix} 15{,}6 \\ 1 \end{vmatrix}$ geeft $\begin{cases} 15{,}6x + 15{,}6y = 234 \\ 15{,}6x + 16{,}8y = 246 \end{cases}$ −

$-1{,}2y = -12$

$\left.\begin{array}{r} y = 10 \\ x + y = 15 \end{array}\right\}$ $x + 10 = 15$
$x = 5$

Er zijn 5 jongens op de verjaardag.

bladzijde 137

19 $\begin{cases} 2x + 3y = 12 \\ y = 4x - 10 \end{cases}$ ofwel

$\begin{cases} 2x + 3y = 12 \\ -4x + y = -10 \end{cases} \begin{vmatrix} 1 \\ 3 \end{vmatrix}$ geeft $\begin{cases} 2x + 3y = 12 \\ -12x + 3y = -30 \end{cases}$ −

$14x = 42$

$\left.\begin{array}{r} x = 3 \\ y = 4x - 10 \end{array}\right\}$ $y = 4 \cdot 3 - 10 = 2$

Het snijpunt is (3, 2).

20 **a** $\begin{cases} 2x + 2y = 9 \\ y = 4x - 3 \end{cases}$ substitueren geeft $2x + 2(4x - 3) = 9$
$2x + 8x - 6 = 9$
$10x = 15$

$\left.\begin{array}{r} x = 1\frac{1}{2} \\ y = 4x - 3 \end{array}\right\}$ $y = 4 \cdot 1\frac{1}{2} - 3 = 3$

De oplossing is $(1\frac{1}{2}, 3)$.

b $\begin{cases} y = \frac{1}{2}x + 1 \\ 3x + 6y = 8 \end{cases}$ substitueren geeft $3x + 6(\frac{1}{2}x + 1) = 8$
$3x + 3x + 6 = 8$
$6x = 2$

$\left.\begin{array}{r} x = \frac{1}{3} \\ y = \frac{1}{2}x + 1 \end{array}\right\}$ $y = \frac{1}{2} \cdot \frac{1}{3} + 1 = 1\frac{1}{6}$

De oplossing is $(\frac{1}{3}, 1\frac{1}{6})$.

c $\begin{cases} x = 5y - 3 \\ 3x + 4y = 29 \end{cases}$ substitueren geeft $3(5y - 3) + 4y = 29$

$15y - 9 + 4y = 29$

$19y = 38$

$\left.\begin{array}{l} y = 2 \\ x = 5y - 3 \end{array}\right\} x = 5 \cdot 2 - 3 = 7$

De oplossing is (7, 2).

21 **a** $\begin{cases} y = x - 4 \\ 6x + 4y = 19 \end{cases}$ substitueren geeft $6x + 4(x - 4) = 19$

$6x + 4x - 16 = 19$

$10x = 35$

$\left.\begin{array}{l} x = 3\frac{1}{2} \\ y = x - 4 \end{array}\right\} y = 3\frac{1}{2} - 4 = -\frac{1}{2}$

De oplossing is $(3\frac{1}{2}, -\frac{1}{2})$.

b $\begin{cases} 7x + 5y = 56 \\ x = 1\frac{1}{2}y + 8 \end{cases}$ substitueren geeft $7(1\frac{1}{2}y + 8) + 5y = 56$

$10\frac{1}{2}y + 56 + 5y = 56$

$15\frac{1}{2}y = 0$

$\left.\begin{array}{l} y = 0 \\ x = 1\frac{1}{2}y + 8 \end{array}\right\} x = 1\frac{1}{2} \cdot 0 + 8 = 8$

De oplossing is (8, 0).

c $\begin{cases} x = 1\frac{1}{2}y + 20\frac{1}{2} \\ 3x + 2y = 1 \end{cases}$ substitueren geeft $3(1\frac{1}{2}y + 20\frac{1}{2}) + 2y = 1$

$4\frac{1}{2}y + 61\frac{1}{2} + 2y = 1$

$6\frac{1}{2}y = -60\frac{1}{2}$

$\left.\begin{array}{l} y = -9\frac{4}{13} \\ x = 1\frac{1}{2}y + 20\frac{1}{2} \end{array}\right\} x = 1\frac{1}{2} \cdot -9\frac{4}{13} + 20\frac{1}{2} = 6\frac{7}{13}$

De oplossing is $(6\frac{7}{13}, -9\frac{4}{13})$.

4.2 Kwadratische formules opstellen

bladzijde 139

22 **a** (3, 5) invullen geeft $3^2 + 2 \cdot 3 + c = 5$

$9 + 6 + c = 5$

$15 + c = 5$

b $c = -10$

23 **a** $x = 5$ en $y = 17$ invullen geeft $2 \cdot 5^2 + b \cdot 5 + 7 = 17$

$50 + 5b + 7 = 17$

$5b = -40$

$b = -8$

b $x = -2$ en $y = 8$ invullen geeft $a \cdot (-2)^2 - 3 \cdot -2 + 5 = 8$

$4a + 6 + 5 = 8$

$4a = -3$

$a = -\frac{3}{4}$

24 **a** $x = 2$ en $y = 0$ invullen geeft $\frac{1}{4} \cdot 2^2 - 2 \cdot 2 + c = 0$

$1 - 4 + c = 0$

$c = 3$

b $c = 3$ geeft $y = \frac{1}{4}x^2 - 2x + 3$

Snijden met de x-as, dus $y = 0$ geeft $\frac{1}{4}x^2 - 2x + 3 = 0$ $\times 4$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2 \vee x = 6$

Het andere snijpunt van de parabool met de x-as is (6, 0).

c $x_{top} = -\frac{b}{2a} = -\frac{-2}{2 \cdot \frac{1}{4}} = 4$

$y_{top} = \frac{1}{4} \cdot 4^2 - 2 \cdot 4 + 3 = -1$

De top is (4, –1).

bladzijde 140

25 **a** $x = 3$ en $y = -20$ invullen geeft $a \cdot 3^2 - 5 \cdot 3 + 4 = -20$

$9a - 15 + 4 = -20$

$9a = -9$

$a = -1$

Dus $y = -x^2 - 5x + 4$.

$x_{top} = -\frac{b}{2a} = -\frac{-5}{2 \cdot -1} = -2\frac{1}{2}$

$y_{top} = -(-2\frac{1}{2})^2 - 5 \cdot -2\frac{1}{2} + 4 = 10\frac{1}{4}$

De top is $(-2\frac{1}{2}, 10\frac{1}{4})$.

b De optie zero (TI) of ROOT (Casio) geeft de nulpunten –5,7 en 0,7.

26 **a** $x = 60$ en $h = 0$ invullen geeft $-0{,}02 \cdot 60^2 + b \cdot 60 = 0$

$-72 + 60b = 0$

$60b = 72$

$b = 1{,}2$

b $b = 1{,}2$ geeft $y = -0{,}02x^2 + 1{,}2x$.

$x_{top} = -\frac{b}{2a} = -\frac{1{,}2}{2 \cdot -0{,}02} = 30$

$h_{top} = -0{,}02 \cdot 30^2 + 1{,}2 \cdot 30 = 18$

De maximale hoogte is 18 meter.

27 **a** $y = x^2 \xrightarrow{\text{4 naar rechts en 5 omhoog}} y = (x - 4)^2 + 5$

top (0, 0) top(4, 5)

b De top is (–3, – 4).

c De top is (3, 6).

bladzijde 141

28 **a** $y = a(x - 3)^2 - 5$

Door (1, –3), dus $a \cdot (1 - 3)^2 - 5 = -3$

$a \cdot (-2)^2 - 5 = -3$

$4a - 5 = -3$

$4a = 2$

$a = \frac{1}{2}$

b $y = a(x - 3)^2 - 5$

Door (3, –5), dus $a \cdot (3 - 3)^2 - 5 = -5$

$a \cdot 0^2 - 5 = -5$

$0 - 5 = -5$

$0 = 0$

Door het punt (3, –5) te gebruiken onstaat voor elke *a* een kloppende vergelijking en is *a* dus niet te berekenen.

29 De top van p_1 is (–2, –1), dus $y = a(x + 2)^2 - 1$.

Door (0, 1), dus $a \cdot (0 + 2)^2 - 1 = 1$

$a \cdot 2^2 - 1 = 1$

$4a = 2$

$a = \frac{1}{2}$

Dus p_1: $y = \frac{1}{2}(x + 2)^2 - 1$.

De top van p_2 is (2, 1) dus $y = a(x-2)^2 + 1$.
Door (3, –1), dus $a \cdot (3-2)^2 + 1 = -1$
$a \cdot 1^2 + 1 = -1$
$a = -2$
Dus p_2: $y = -2(x-2)^2 + 1$.

30 **a** De top is (2, 6) dus $y = a(x-2)^2 + 6$.
Door (4, 4), dus $a \cdot (4-2)^2 + 6 = 4$
$a \cdot 2^2 + 6 = 4$
$4a = -2$
$a = -\frac{1}{2}$
Dus $y = -\frac{1}{2}(x-2)^2 + 6$.

b $-\frac{1}{2}(x-2)^2 + 6 = -\frac{1}{2}(x^2 - 4x + 4) + 6 = -\frac{1}{2}x^2 + 2x - 2 + 6 = -\frac{1}{2}x^2 + 2x + 4$
Dus $y = -\frac{1}{2}x^2 + 2x + 4$.

31 De top van p_1 is (–2, 1), dus $y = a(x+2)^2 + 1$.
Door (0, –1), dus $a \cdot (0+2)^2 + 1 = -1$
$a \cdot 2^2 + 1 = -1$
$4a = -2$
$a = -\frac{1}{2}$
Een formule is $y = -\frac{1}{2}(x+2)^2 + 1$.
$-\frac{1}{2}(x+2)^2 + 1 = -\frac{1}{2}(x^2 + 4x + 4) + 1 = -\frac{1}{2}x^2 - 2x - 2 + 1 = -\frac{1}{2}x^2 - 2x - 1$
Dus p_1: $y = -\frac{1}{2}x^2 - 2x - 1$.

De top van p_2 is (3, 0), dus $y = a(x-3)^2$.
Door (0, 3), dus $a \cdot (0-3)^2 = 3$
$a \cdot (-3)^2 = 3$
$9a = 3$
$a = \frac{1}{3}$
Een formule is $y = \frac{1}{3}(x-3)^2$.
$\frac{1}{3}(x-3)^2 = \frac{1}{3}(x^2 - 6x + 9) = \frac{1}{3}x^2 - 2x + 3$
Dus p_2: $y = \frac{1}{3}x^2 - 2x + 3$.

De top van p_3 is (0, 3), dus $y = ax^2 + 3$.
Door (1,–1), dus $a \cdot (1)^2 + 3 = -1$
$a = -4$
Dus p_3: $y = -4x^2 + 3$.

De top van p_4 is (2, 2), dus $y = a(x-2)^2 + 2$.
Door (1, 3), dus $a \cdot (1-2)^2 + 2 = 3$
$a \cdot (-1)^2 + 2 = 3$
$a = 1$
Een formule is $y = (x-2)^2 + 2$.
$(x-2)^2 + 2 = x^2 - 4x + 4 + 2 = x^2 - 4x + 6$
Dus p_4: $y = x^2 - 4x + 6$.

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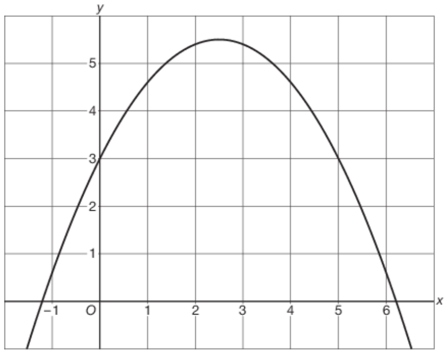
32 De top is (–3, 4), dus $y = a(x+3)^2 + 4$.
Door (–1, 0), dus $a \cdot (-1+3)^2 + 4 = 0$
$a \cdot 2^2 + 4 = 0$
$4a = -4$
$a = -1$
Dus $y = -(x+3)^2 + 4$.
$-(x+3)^2 + 4 = -(x^2 + 6x + 9) + 4 = -x^2 - 6x - 9 + 4 = -x^2 - 6x - 5$.
Dus $y = -x^2 - 6x - 5$.

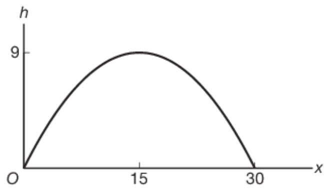
33 Zie de grafiek hiernaast.
De top is $(2\frac{1}{2}, 5\frac{1}{2})$, dus $y = a(x - 2\frac{1}{2})^2 + 5\frac{1}{2}$.
Door (0, 3), dus $a \cdot (0 - 2\frac{1}{2})^2 + 5\frac{1}{2} = 3$
$a \cdot (2\frac{1}{2})^2 + 5\frac{1}{2} = 3$
$6\frac{1}{4}a = -2\frac{1}{2}$
$a = -\frac{2}{5}$
Dus $y = -\frac{2}{5}(x - 2\frac{1}{2})^2 + 5\frac{1}{2}$.
$x = 10$ geeft $y = -\frac{2}{5}(10 - 2\frac{1}{2})^2 + 5\frac{1}{2} = -17$.
Dus (10, –17) ligt op de parabool.



34 **a** $p = 10$ en $q = 6$
b $x = 0$ geeft $y = a \cdot 0^2 + b \cdot 0 = 0$.
Dus de parabool gaat door de oorsprong.
$p = 10$ en $q = 6$ geeft $y = a(x - 10)^2 + 6$.
Door (0, 0), dus $a \cdot (0 - 10)^2 + 6 = 0$
$a \cdot (-10)^2 + 6 = 0$
$100a + 6 = 0$
$100a = -6$
$a = -\frac{3}{50}$
c $a = -\frac{3}{50}$ geeft $y = -\frac{3}{50}(x - 10)^2 + 6$.
$-\frac{3}{50}(x-10)^2 + 6 = -\frac{3}{50}(x^2 - 20x + 100) + 6 = -\frac{3}{50}x^2 + 1\frac{1}{5}x - 6 + 6 = -\frac{3}{50}x^2 + 1\frac{1}{5}$
Dus $b = 1\frac{1}{5}$.

35 De top is (2,4; 28,8), dus $h = a(t - 2{,}4)^2 + 28{,}8$.
Door (0, 0), dus $a \cdot (0 - 2{,}4)^2 + 28{,}8 = 0$
$5{,}76a = -28{,}8$
$a = -5$
Dus $h = -5(t - 2{,}4)^2 + 28{,}8$.
$-5(t - 2{,}4)^2 + 28{,}8 = -5(t^2 - 4{,}8t + 5{,}76) + 28{,}8 = -5t^2 + 24t - 28{,}8 + 28{,}8 = -5t^2 + 24t$
Dus $a = -5$ en $b = 24$.

36 De top is (15, 9), dus $h = a \cdot (x - 15)^2 + 9$.
Door (0, 0) dus $a \cdot (0 - 15)^2 + 9 = 0$
$225a = -9$
$a = -\frac{1}{25}$
Dus $h = -\frac{1}{25}(x - 15)^2 + 9$.
$-\frac{1}{25}(x-15)^2 + 9 = -\frac{1}{25}(x^2 - 30x + 225) + 9 =$
$-\frac{1}{25}x^2 + 1\frac{1}{5}x - 9 + 9 = -\frac{1}{25}x^2 + 1\frac{1}{5}x$
Dus $a = -\frac{1}{25}$ en $b = 1\frac{1}{5}$.



37 **a** $y = ax^2 - 6x + c$
Door (1, –1), dus $a \cdot 1^2 - 6 \cdot 1 + c = -1$
$a - 6 + c = -1$
b $y = ax^2 - 6x + c$
Door (4, 11), dus $a \cdot 4^2 - 6 \cdot 4 + c = 11$
$16a - 24 + c = 11$

38 (1, 8) invullen geeft $a \cdot 1^2 + c = 8$, ofwel $a + c = 8$.
(2, 17) invullen geeft $a \cdot 2^2 + c = 17$, ofwel $4a + c = 17$.

$$\begin{cases} a + c = 8 \\ 4a + c = 17 \end{cases}$$
$$-3a = -9$$
$$\left.\begin{array}{r} a = 3 \\ a + c = 8 \end{array}\right\} \begin{array}{l} 3 + c = 8 \\ c = 5 \end{array}$$

Dus $y = 3x^2 + 5$.

39 (1, 9) invullen geeft $-1^2 + b \cdot 1 + c = 9$, ofwel $b + c = 10$.
(4, 6) invullen geeft $-4^2 + b \cdot 4 + c = 6$, ofwel $4b + c = 22$.

$$\begin{cases} b + c = 10 \\ 4b + c = 22 \end{cases}$$
$$-3b = -12$$
$$\left.\begin{array}{r} b = 4 \\ b + c = 10 \end{array}\right\} \begin{array}{l} 4 + c = 10 \\ c = 6 \end{array}$$

Dus $y = -x^2 + 4x + 6$.

40 **a** (−1, 5) invullen geeft $2 \cdot (-1)^2 + b \cdot -1 + c = 5$, ofwel $-b + c = 3$.
(3, −7) invullen geeft $2 \cdot 3^2 + b \cdot 3 + c = -7$, ofwel $3b + c = -25$.

$$\begin{cases} -b + c = 3 \\ 3b + c = -25 \end{cases}$$
$$-4b = 28$$
$$\left.\begin{array}{r} b = -7 \\ -b + c = 3 \end{array}\right\} \begin{array}{l} --7 + c = 3 \\ 7 + c = 3 \\ c = -4 \end{array}$$

Dus $b = -7$ en $c = -4$.

b $b = -7$ en $c = -4$ geeft $y = 2x^2 - 7x - 4$.
Snijden met de x-as, dus $y = 0$ geeft $2x^2 - 7x - 4 = 0$

$$D = (-7)^2 - 4 \cdot 2 \cdot -4 = 81, \text{ dus } \sqrt{D} = \sqrt{81} = 9$$
$$x = \frac{7-9}{4} = -\tfrac{1}{2} \vee x = \frac{7+9}{4} = 4$$

De snijpunten met de x-as zijn $(-\tfrac{1}{2}, 0)$ en (4, 0).

41 (1, 0) invullen geeft $a(1-3)^2 + q = 0$, ofwel $4a + q = 0$.
(4, 6) invullen geeft $a(4-3)^2 + q = 6$, ofwel $a + q = 6$.

$$\begin{array}{l} 4a + q = 0 \\ a + q = 6 \end{array}$$
$$3a = -6$$
$$\left.\begin{array}{r} a = -2 \\ a + q = 6 \end{array}\right\} \begin{array}{l} -2 + q = 6 \\ q = 8 \end{array}$$

$a = -2$ en $q = 8$ geeft $y = -2(x+1)^2 + 8$.
Dus de y-coördinaat van de top van de parabool is 8.

42 (−4, 20) invullen geeft $a(-4+1)^2 + q = 20$, ofwel $9a + q = 20$.
$(\tfrac{1}{2}, -7)$ invullen geeft $a(\tfrac{1}{2}+1)^2 + q = -7$, ofwel $2\tfrac{1}{4}a + q = -7$.

$$\begin{cases} 9a + q = 20 \\ 2\tfrac{1}{4}a + q = -7 \end{cases}$$
$$6\tfrac{3}{4}a = 27$$
$$\left.\begin{array}{r} a = 4 \\ 9a + q = 20 \end{array}\right\} \begin{array}{l} 9 \cdot 4 + q = 20 \\ 36 + q = 20 \\ q = -16 \end{array}$$

$a = 4$ en $q = -16$ geeft $y = 4(x+1)^2 - 16$.

Snijden met de x-as, dus $y = 0$ geeft

$$4(x+1)^2 - 16 = 0$$
$$4(x^2 + 2x + 1) - 16 = 0$$
$$4x^2 + 8x + 4 - 16 = 0$$
$$4x^2 + 8x - 12 = 0 \quad :4$$
$$x^2 + 2x - 3 = 0$$
$$(x-1)(x+3) = 0$$
$$x = 1 \vee x = -3$$

Dus de snijpunten met de x-as zijn $(-3, 0)$ en $(1, 0)$.

43 **a** $(2, -1)$ invullen in $y = x^2 + px + q$ geeft $2^2 + p \cdot 2 + q = -1$, ofwel $2p + q = -5$.
$(2, -1)$ invullen in $y = 2px - q$ geeft $2p \cdot 2 - q = -1$, ofwel $4p - q = -1$.

$$\begin{cases} 2p + q = -5 \\ 4p - q = -1 \end{cases}$$
$$6p = -6 \quad (+)$$
$$p = -1$$
$$2p + q = -5 \quad \} \quad 2 \cdot -1 + q = -5$$
$$-2 + q = -5$$
$$q = -3$$

Dus $p = -1$ en $q = -3$.

b De parabool $y = x^2 - x - 3$ snijden met de lijn $y = -2x + 3$ geeft
$x^2 - x - 3 = -2x + 3$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$x = -3 \vee x = 2$
$x = -3$ invullen in $y = -2x + 3$ geeft $y = -2 \cdot -3 + 3 = 9$.
Het andere snijpunt is $(-3, 9)$.

44 De parabool $y = ax^2 + bx + c$ snijdt de y-as in het punt $(0, 4)$.
Hieruit volgt direct $c = 4$.
Dus $y = ax^2 + bx + 4$.
$(-2, -10)$ invullen geeft $a \cdot (-2)^2 + b \cdot -2 + 4 = -10$, ofwel $4a - 2b = -14$.
$(3, 5)$ invullen geeft $a \cdot 3^2 + b \cdot 3 + 4 = 5$, ofwel $9a + 3b = 1$.

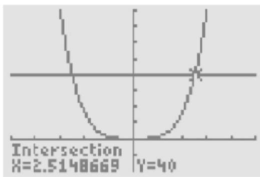
$$\begin{cases} 4a - 2b = -14 \\ 9a + 3b = 1 \end{cases} \begin{vmatrix} 3 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 12a - 6b = -42 \\ 18a + 6b = 2 \end{cases}$$
$$30a = -40 \quad (+)$$
$$a = -1\tfrac{1}{3}$$
$$9a + 3b = 1 \quad \} \quad 9 \cdot -1\tfrac{1}{3} + 3b = 1$$
$$-12 + 3b = 1$$
$$3b = 13$$
$$b = 4\tfrac{1}{3}$$

Dus $y = -1\frac{1}{3}x^2 + 4\frac{1}{3}x + 4$.

4.3 Hogeregraadsvergelijkingen

bladzijde 145

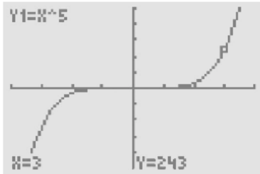
45 **a** Voer in $y_1 = x^4$. Zie het GR-scherm hiernaast.



b Voer in $y_2 = 40$.
Er zijn twee snijpunten, de vergelijking heeft dus twee oplossingen.
De optie intersect geeft $x \approx -2{,}51$ en $x \approx 2{,}51$.

c De vergelijking $x^4 = -40$ heeft geen oplossingen want x^4 is altijd groter dan of gelijk aan 0.

46 **a** Voer in $y_1 = x^5$. Zie het GR-scherm hiernaast.
b Voer in $y_2 = 250$.
De vergelijking $x^5 = 250$ heeft één oplossing want de grafieken van y_1 en y_2 hebben één snijpunt.
c Voer in $y_3 = -250$.
De vergelijking $x^5 = -250$ heeft één oplossing want de grafieken van y_1 en y_3 hebben één snijpunt.



bladzijde 146

47 **a** $x^6 = 20$
$x = \sqrt[6]{20} \vee x = -\sqrt[6]{20}$

b $5x^3 = 100$
$x^3 = 20$
$x = \sqrt[3]{20}$

c $x^2 + 7 = 18$
$x^2 = 11$
$x = \sqrt{11} \vee x = -\sqrt{11}$

d $3x^7 + 25 = 4$
$3x^7 = -21$
$x^7 = -7$
$x = \sqrt[7]{-7}$

e $\frac{1}{2}x^6 + 12 = 9$
$\frac{1}{2}x^6 = -3$
$x^6 = -6$
geen oplossing

f $0{,}3x^8 + 5 = 11$
$0{,}3x^8 = 6$
$x^8 = 20$
$x = \sqrt[8]{20} \vee x = -\sqrt[8]{20}$

bladzijde 147

48 **a** $3x^5 + 10 = 16$
$3x^5 = 6$
$x^5 = 2$
$x = \sqrt[5]{2}$
$x \approx 1{,}15$

b $2x^5 + 9 = 1$
$2x^5 = -8$
$x^5 = -4$
$x = \sqrt[5]{-4}$
$x \approx -1{,}32$

c $3x^4 - 5 = 10$
$3x^4 = 15$
$x^4 = 5$
$x = \sqrt[4]{5} \vee x = -\sqrt[4]{5}$
$x \approx 1{,}50 \vee x \approx -1{,}50$

d $3x^4 + 10 = 4$
$3x^4 = -6$
$x^4 = -2$
geen oplossing

e $\frac{1}{3}x^6 + 2 = 6$
$\frac{1}{3}x^6 = 4$
$x^6 = 12$
$x = \sqrt[6]{12} \vee x = -\sqrt[6]{12}$
$x \approx 1{,}51 \vee x \approx -1{,}51$

f $-\frac{1}{2}x^6 + 6 = 2$
$-\frac{1}{2}x^6 = -4$
$x^6 = 8$
$x = \sqrt[6]{8} \vee x = -\sqrt[6]{8}$
$x \approx 1{,}41 \vee x \approx -1{,}41$

49 **a** $4^3 = 64$ dus $\sqrt[3]{64} = 4$
b $x = \sqrt[3]{125} = 5$

c

x	x^2	x^3	x^4	x^5	x^6
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	×
5	25	125	625	×	×
6	36	216	×	×	×
7	49	343	×	×	×
8	64	×	×	×	×
9	81	×	×	×	×

bladzijde 148

50

a $\frac{1}{2}x^3 - 8 = 100$
$\frac{1}{2}x^3 = 108$
$x^3 = 216$
$x = 6$

b $\frac{1}{9}x^6 - 1 = 80$
$\frac{1}{9}x^6 = 81$
$x^6 = 729$
$x = 3 \vee x = -3$

c $82 - \frac{1}{3}x^5 = 1$
$-\frac{1}{3}x^5 = -81$
$x^5 = 243$
$x = 3$

d $3(2x - 1)^2 = 147$
$(2x - 1)^2 = 49$
$2x - 1 = 7 \vee 2x - 1 = -7$
$2x = 8 \vee 2x = -6$
$x = 4 \vee x = -3$

e $5(x + 2)^3 - 36 = 99$
$5(x + 2)^3 = 135$
$(x + 2)^3 = 27$
$x + 2 = 3$
$x = 1$

f $\frac{1}{5}(4x + 1)^4 - 25 = 100$
$\frac{1}{5}(4x + 1)^4 = 125$
$(4x + 1)^4 = 625$
$4x + 1 = 5 \vee 4x + 1 = -5$
$4x = 4 \vee 4x = -6$
$x = 1 \vee x = -1\frac{1}{2}$

51

a $x^3 - x^2 - 6x = x(x^2 - x - 6) = x(x + 2)(x - 3)$

b $x^3 - x^2 - 6x = 0$
$x(x + 2)(x - 3) = 0$
$x = 0 \vee x + 2 = 0 \vee x - 3 = 0$
$x = 0 \vee x = -2 \vee x = 3$

52

a $x^4 - x^2 - 6 = 0$ is te schrijven als $(x^2)^2 - x^2 - 6 = 0$.
$x^2 = u$ geeft vervolgens $u^2 - u - 6 = 0$.

b $u^2 - u - 6 = 0$
$(u + 2)(u - 3) = 0$
$u + 2 = 0 \vee u - 3 = 0$
$u = -2 \vee u = 3$

c $x^2 = u$ en $u = -2$ geeft $x^2 = -2$, dus geen oplossing.
$x^2 = u$ en $u = 3$ geeft $x^2 = 3$, dus $x = \sqrt{3}$ en $x = -\sqrt{3}$.
Dus de vergelijking $x^4 - x^2 - 6 = 0$ heeft de oplossingen $x = \sqrt{3}$ en $x = -\sqrt{3}$.

53 **a** $x^3 - 5x^2 + 6x = 0$
$x(x^2 - 5x + 6) = 0$
$x(x-2)(x-3) = 0$
$x = 0 \lor x = 2 \lor x = 3$

b $x^3 - 5x^2 = 6x$
$x^3 - 5x^2 - 6x = 0$
$x(x^2 - 5x - 6) = 0$
$x(x+1)(x-6) = 0$
$x = 0 \lor x = -1 \lor x = 6$

c $x^3 = 4x^2 + 12x$
$x^3 - 4x^2 - 12x = 0$
$x(x^2 - 4x - 12) = 0$
$x(x+2)(x-6) = 0$
$x = 0 \lor x = -2 \lor x = 6$

d $x^3 + x^2 = 12x$
$x^3 + x^2 - 12x = 0$
$x(x^2 + x - 12) = 0$
$x(x-3)(x+4) = 0$
$x = 0 \lor x = 3 \lor x = -4$

54 **a** $x^4 - 10x^2 + 9 = 0$
Stel $x^2 = u$.
$u^2 - 10u + 9 = 0$
$(u-1)(u-9) = 0$
$u = 1 \lor u = 9$
$x^2 = 1 \lor x^2 = 9$
$x = 1 \lor x = -1 \lor x = 3 \lor x = -3$

b $x^4 - 8x^2 - 9 = 0$
Stel $x^2 = u$.
$u^2 - 8u - 9 = 0$
$(u+1)(u-9) = 0$
$u = -1 \lor u = 9$
$x^2 = -1 \lor x^2 = 9$
$x = 3 \lor x = -3$

c $x^4 + 16 = 10x^2$
$x^4 - 10x^2 + 16 = 0$
Stel $x^2 = u$.
$u^2 - 10u + 16 = 0$
$(u-2)(u-8) = 0$
$u = 2 \lor u = 8$
$x^2 = 2 \lor x^2 = 8$
$x = \sqrt{2} \lor x = -\sqrt{2} \lor x = \sqrt{8} \lor x = -\sqrt{8}$

d $x^4 + x^2 = 12$
$x^4 + x^2 - 12 = 0$
Stel $x^2 = u$.
$u^2 + u - 12 = 0$
$(u-3)(u+4) = 0$
$u = 3 \lor u = -4$
$x^2 = 3 \lor x^2 = -4$
$x = \sqrt{3} \lor x = -\sqrt{3}$

55 **a** $x^3 + 25x = 10x^2$
$x^3 - 10x^2 + 25x = 0$
$x(x^2 - 10x + 25) = 0$
$x(x-5)(x-5) = 0$
$x = 0 \lor x = 5$

b $x^4 - 13x^2 + 36 = 0$
Stel $x^2 = u$.
$u^2 - 13u + 36 = 0$
$(u-4)(u-9) = 0$
$u = 4 \lor u = 9$
$x^2 = 4 \lor x^2 = 9$
$x = 2 \lor x = -2 \lor x = 3 \lor x = -3$

c $x^4 - 16x^3 = 36x^2$
$x^4 - 16x^3 - 36x^2 = 0$
$x^2(x^2 - 16x - 36) = 0$
$x^2(x+2)(x-18) = 0$
$x = 0 \lor x = -2 \lor x = 18$

d $x^3 + 6x^2 = 16x$
$x^3 + 6x^2 - 16x = 0$
$x(x^2 + 6x - 16) = 0$
$x(x-2)(x+8) = 0$
$x = 0 \lor x = 2 \lor x = -8$

4.4 Ongelijkheden oplossen

56 **a** $x^2 - 3x = 2x - 4$
$x^2 - 5x + 4 = 0$
$(x-1)(x-4) = 0$
$x = 1 \lor x = 4$

b $x^2 - 3x < 2x - 4$ geeft $1 < x < 4$.

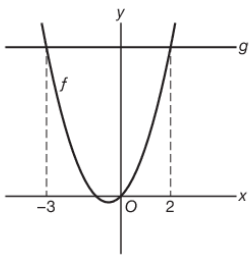
57 **a** $\underbrace{x^2+x}_{f(x)} > \underbrace{6}_{g(x)}$

$x^2+x=6$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3 \vee x=2$



$x^2+x>6$ geeft $x<-3 \vee x>2$.

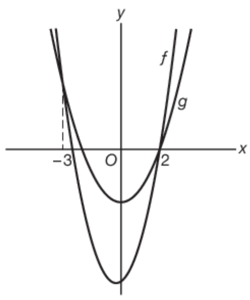
c $\underbrace{2x^2+x-10}_{f(x)} > \underbrace{x^2-4}_{g(x)}$

$2x^2+x-10=x^2-4$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3 \vee x=2$



$2x^2+x-10>x^2-4$ geeft $x<-3 \vee x>2$.

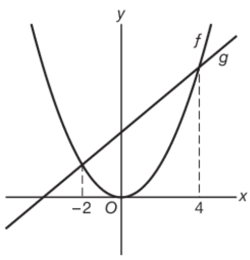
b $\underbrace{x^2}_{f(x)} < \underbrace{2x+8}_{g(x)}$

$x^2=2x+8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x=4 \vee x=-2$



$x^2<2x+8$ geeft $-2<x<4$.

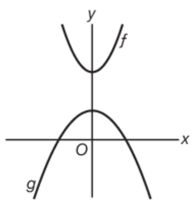
d $\underbrace{2x^2+7}_{f(x)} < \underbrace{3-x^2}_{g(x)}$

$2x^2+7=3-x^2$

$3x^2=-4$

$x^2=-\frac{4}{3}$

geen oplossing

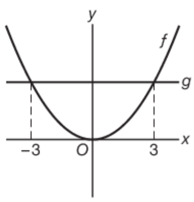


$2x^2+7<3-x^2$ geeft geen oplossingen.

58 **a** $\underbrace{x^4}_{f(x)} > \underbrace{81}_{g(x)}$

$x^4=81$

$x=3 \vee x=-3$

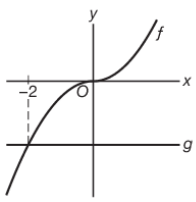


$x^4>81$ geeft $x<-3 \vee x>3$.

b $\underbrace{x^3}_{f(x)} < \underbrace{-8}_{g(x)}$

$x^3=-8$

$x=-2$



$x^3<-8$ geeft $x<-2$.

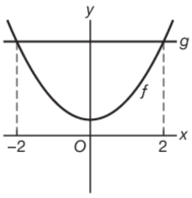
c $\underbrace{\tfrac{1}{2}x^4 + 1}_{f(x)} < \underbrace{9}_{g(x)}$

$\tfrac{1}{2}x^4 + 1 = 9$

$\tfrac{1}{2}x^4 = 8$

$x^4 = 16$

$x = 2 \vee x = -2$



$\tfrac{1}{2}x^4 + 1 < 9$ geeft $-2 < x < 2$.

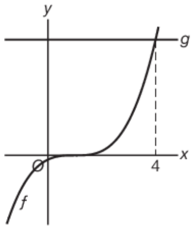
d $\underbrace{\tfrac{1}{3}(x-1)^3}_{f(x)} > \underbrace{9}_{g(x)}$

$\tfrac{1}{3}(x-1)^3 = 9$

$(x-1)^3 = 27$

$x - 1 = 3$

$x = 4$



$\tfrac{1}{3}(x-1)^3 > 9$ geeft $x > 4$.

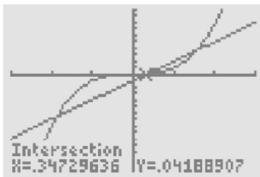
59 $x^4 = x^3 + 1$ geeft $x^4 - x^3 - 1 = 0$.
Het lukt niet deze hogeregraadsvergelijking op te lossen, want je kunt niet ontbinden in factoren en je kunt niet gebruik maken van de substitutie $x^2 = u$.

bladzijde 153

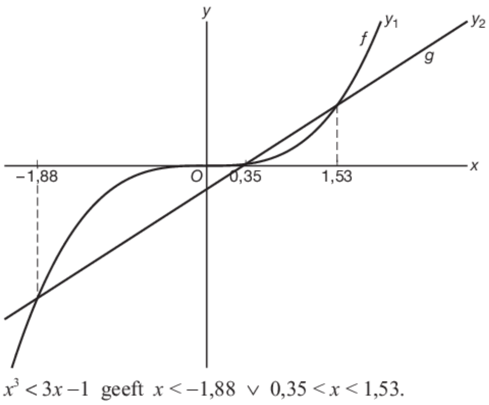
60 **a** Voer in $y_1 = x^3 - 5x - 2$.
De optie zero (TI) of ROOT (Casio) geeft
$x = -2 \vee x \approx -0{,}414 \vee x \approx 2{,}414$.
b Voer in $y_1 = -0{,}5x^3 + 2x^2 - 2$.
De optie zero (TI) of ROOT (Casio) geeft
$x \approx -0{,}903 \vee x \approx 1{,}194 \vee x \approx 3{,}709$.
c Voer in $y_1 = -x^3 + 6x$ en $y_2 = 0{,}4x^2 + 2$.
De optie intersect geeft $x \approx -2{,}799 \vee x \approx 0{,}348 \vee x \approx 2{,}050$.
d Voer in $y_1 = x^3 - 3$ en $y_2 = 0{,}5x^2 - 2x$.
De optie intersect geeft $x \approx 1{,}116$.

61 **a** Voer in $y_1 = 0{,}2x^3 - 3x + 2$.
De optie zero (TI) of ROOT (Casio) geeft de nulpunten $-4{,}17$, $0{,}69$ en $3{,}48$.
b Voer in $y_1 = -0{,}4x^4 + 2x^3 - 8x + 5$.
De optie zero (TI) of ROOT (Casio) geeft de nulpunten $-1{,}95$, $0{,}70$, $2{,}36$ en $3{,}89$.

62 **a** Voer in $y_1 = x^3$ en $y_2 = 3x - 1$.
De optie intersect geeft $x \approx -1{,}88 \vee x \approx 0{,}35 \vee x \approx 1{,}53$.

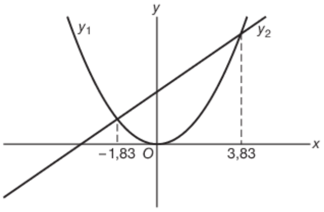


b



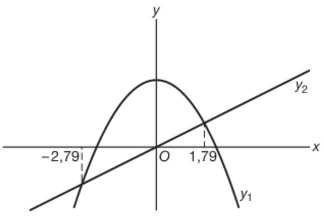
$x^3 < 3x - 1$ geeft $x < -1{,}88 \vee 0{,}35 < x < 1{,}53$.

63 **a** $x^2 < 2x + 7$
Voer in $y_1 = x^2$ en $y_2 = 2x + 7$.
De optie intersect geeft
$x \approx -1{,}83 \vee x \approx 3{,}83$.



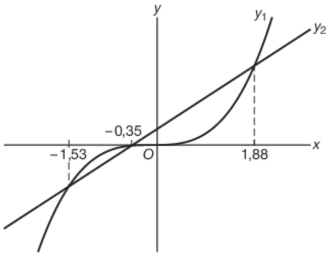
$x^2 < 2x + 7$ geeft $-1{,}83 < x < 3{,}83$.

b $5 - x^2 < x$
Voer in $y_1 = 5 - x^2$ en $y_2 = x$.
De optie intersect geeft
$x \approx -2{,}79 \vee x \approx 1{,}79$.



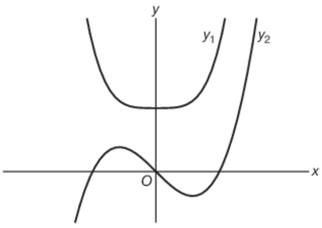
$5 - x^2 < x$ geeft $x < -2{,}79 \vee x > 1{,}79$.

c $x^3 > 3x + 1$
Voer in $y_1 = x^3$ en $y_2 = 3x + 1$.
De optie intersect geeft
$x \approx -1{,}53 \vee x \approx -0{,}35 \vee x \approx 1{,}88$.



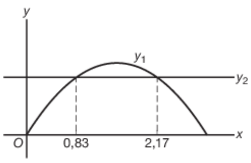
$x^3 > 3x + 1$ geeft
$-1{,}53 < x < -0{,}35 \vee x > 1{,}88$.

d $x^4 + 1 > x^3 - x$
Voer in $y_1 = x^4 + 1$ en $y_2 = x^3 - x$.
De optie intersect geeft geen oplossingen.



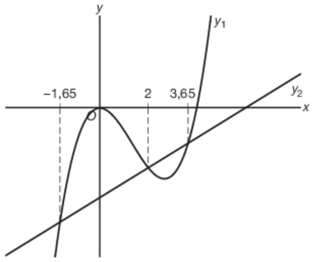
$x^4 + 1 > x^3 - x$ geeft elke x is oplossing.

64 $h > 9$ geeft $-5t^2 + 15t > 9$
Voer in $y_1 = -5x^2 + 15x$ en $y_2 = 9$.
De optie intersect geeft $x \approx 0{,}83 \vee x \approx 2{,}17$.



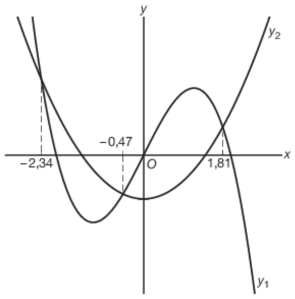
De bal is $2{,}17 - 0{,}83 \approx 1{,}3$ seconde hoger dan 9 m.

65 **a** $x^3 - 4x^2 < 2x - 12$
Voer in $y_1 = x^3 - 4x^2$ en $y^2 = 2x - 12$.
De optie intersect geeft
$x \approx -1{,}65 \ \vee \ x = 2 \ \vee \ x \approx 3{,}65$.



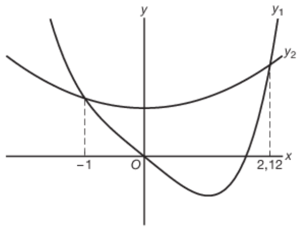
$x^3 - 4x^2 < 2x - 12$ geeft
$x < -1{,}65 \ \vee \ 2 < x < 3{,}65$.

b $4x - x^3 > x^2 - 2$
Voer in $y_1 = 4x - x^3$ en $y_2 = x^2 - 2$.
De optie intersect geeft
$x \approx -2{,}34 \ \vee \ x \approx -0{,}47 \ \vee \ x \approx 1{,}81$.



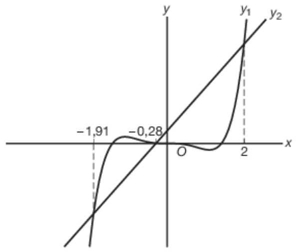
$4x - x^3 > x^2 - 2$ geeft
$x < -2{,}34 \ \vee \ -0{,}47 < x < 1{,}81$.

c $x^4 - 5x < x^2 + 5$
Voer in $y_1 = x^4 - 5x$ en $y_2 = x^2 + 5$.
De optie intersect geeft
$x = -1 \ \vee \ x \approx 2{,}12$.



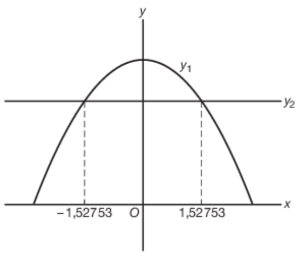
$x^4 - 5x < x^2 + 5$ geeft $-1 < x < 2{,}12$.

d $x^5 - 2x^3 > 7x + 2$
Voer in $y_1 = x^5 - 2x^3$ en $y_2 = 7x + 2$.
De optie intersect geeft
$x \approx -1{,}91 \ \vee \ x \approx -0{,}28 \ \vee \ x = 2$.



$x^5 - 2x^3 > 7x + 2$ geeft
$-1{,}91 < x < -0{,}28 \ \vee \ x > 2$.

66 $h < 3{,}7$ geeft $-0{,}12x^2 + 3{,}98 < 3{,}7$
Voer in $y_1 = -0{,}12x^2 + 3{,}98$ en $y_2 = 3{,}7$.
De optie intersect geeft $x \approx -1{,}52753 \ \vee \ x \approx 1{,}52753$.



$1{,}52753 - -1{,}52753 = 3{,}05506$
Dus de breedte is maximaal 3,05 m.

Diagnostische toets

bladzijde 32

1 **a** Snijden met de x-as, dus $y = 0$ geeft $5x = -17$, dus $x = -3\frac{2}{5}$.
Het snijpunt met de x-as is $(-3\frac{2}{5}, 0)$.
Snijden met de y-as, dus $x = 0$ geeft $3y = -17$, dus $y = -5\frac{2}{3}$.
Het snijpunt met de y-as is $(0, -5\frac{2}{3})$.

b $A(-25, 36)$ invullen bij l geeft $5 \cdot -25 + 3 \cdot 36 = -125 + 108 = -17$.
Klopt, dus A ligt op l.
$B(60, -105)$ invullen bij l geeft $5 \cdot 60 + 3 \cdot -105 = 300 - 315 = -17$.
Klopt niet, dus B ligt niet op l.
$C(83, -144)$ invullen bij l geeft $5 \cdot 83 + 3 \cdot -144 = 415 - 432 = -17$.
Klopt, dus C ligt op l.

c $(-56, q)$ invullen bij l geeft $5 \cdot -56 + 3 \cdot q = -17$
$-280 + 3q = -17$
$3q = 263$
$q = 87\frac{2}{3}$

d $m \mathbin{/\!/} l$, dus m: $5x + 3y = c$
$D(2, -4)$ op m, dus $c = 5 \cdot 2 + 3 \cdot -4 = -2$.
Dus m: $5x + 3y = -2$.

2 **a**
$$\begin{cases} 2x - 5y = -9 \\ 3x + 5y = 24 \end{cases}$$
optellen (+): $5x = 15$
$x = 3$
$x = 3$ invullen in $3x + 5y = 24$: $9 + 5y = 24$
$5y = 15$
$y = 3$
De oplossing is (3, 3).

b
$$\begin{cases} 4x + 2y = 4 \\ 4x - 7y = 40 \end{cases}$$
aftrekken (−): $9y = -36$
$y = -4$
$y = -4$ invullen in $4x + 2y = 4$: $4x - 8 = 4$
$4x = 12$
$x = 3$
De oplossing is (3, −4).

c
$$\begin{cases} 4x + 5y = 27 \\ -2x + 3y = 25 \end{cases} \begin{vmatrix} 1 \\ 2 \end{vmatrix} \text{ geeft } \begin{cases} 4x + 5y = 27 \\ -4x + 6y = 50 \end{cases}$$
optellen (+): $11y = 77$
$y = 7$
$y = 7$ invullen in $4x + 5y = 27$: $4x + 35 = 27$
$4x = -8$
$x = -2$
De oplossing is (−2, 7).

d
$$\begin{cases} 6x - 3y = 19 \\ -4x - 6y = 14 \end{cases} \begin{vmatrix} 2 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 12x - 6y = 38 \\ -4x - 6y = 14 \end{cases}$$
aftrekken (−): $16x = 24$
$x = 1\frac{1}{2}$
$x = 1\frac{1}{2}$ invullen in $6x - 3y = 19$: $9 - 3y = 19$
$-3y = 10$
$y = -3\frac{1}{3}$
De oplossing is $(1\frac{1}{2}, -3\frac{1}{3})$.

3
$$\begin{cases} 5x - 6y = -27 \\ 15x + 8y = 10 \end{cases} \begin{vmatrix} 3 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 15x - 18y = -81 \\ 15x + 8y = 10 \end{cases}$$
aftrekken (−): $-26y = -91$
$y = 3\frac{1}{2}$
$y = 3\frac{1}{2}$ invullen in $5x - 6y = -27$: $5x - 21 = -27$
$5x = -6$
$x = -1\frac{1}{5}$
Het snijpunt is $(-1\frac{1}{5}, 3\frac{1}{2})$.

4 Het totaal aan cijfers in beide klassen is $35 \cdot 6{,}8 = 238$.
Stel het aantal leerlingen in klas 4h1 x en het aantal leerlingen in klas 4h2 y.
Uit de gegevens volgt het stelsel
$$\begin{cases} x + y = 35 \\ 6{,}5x + 7{,}2y = 238 \end{cases} \begin{vmatrix} 6{,}5 \\ 1 \end{vmatrix} \text{ geeft } \begin{cases} 6{,}5x + 6{,}5y = 227{,}5 \\ 6{,}5x + 7{,}2y = 238 \end{cases}$$
aftrekken (−): $-0{,}7y = -10{,}5$
$y = 15$
$y = 15$ invullen in $x + y = 35$: $x + 15 = 35$
$x = 20$
Het aantal leerlingen in klas 4h1 is 20.

5 **a** $\begin{cases} 5x - 3y = 3 \\ y = \frac{2}{3}x - 4 \end{cases}$ substitueren geeft $5x - 3(\frac{2}{3}x - 4) = 3$

$5x - 2x + 12 = 3$
$3x = -9$
$x = -3$
$y = \frac{2}{3}x - 4$ $\} \ y = \frac{2}{3} \cdot -3 - 4 = -6$

De oplossing is (–3, –6)

b $\begin{cases} x = 1{,}4y - 3 \\ -5x + 6y = 8 \end{cases}$ substitueren geeft $-5(1{,}4y - 3) + 6y = 8$

$-7y + 15 + 6y = 8$
$-y = -7$
$y = 7$
$x = 1{,}4y - 3$ $\} \ x = 1{,}4 \cdot 7 - 3 = 6{,}8$

De oplossing is (6,8; 7).

6 **a** (–2, 5) invullen geeft $\frac{1}{2} \cdot (-2)^2 + 3 \cdot -2 + c = 5$

$\frac{1}{2} \cdot 4 - 6 + c = 5$
$2 - 6 + c = 5$
$c = 9$

b (5, 3) invullen geeft $\frac{2}{5} \cdot 5^2 + b \cdot 5 + 6 = 3$

$\frac{2}{5} \cdot 25 + 5b + 6 = 3$
$10 + 5b + 6 = 3$
$5b = -13$
$b = -2\frac{3}{5}$

c (5, –1) invullen geeft $a \cdot 5^2 - 1\frac{1}{5} \cdot 5 + 3 = -1$

$25a - 6 + 3 = -1$
$25a = 2$
$a = \frac{2}{25}$

7 **a** De top is (1, –5), dus $y = a(x-1)^2 - 5$.
Door (6, –15), dus $a \cdot (6-1)^2 - 5 = -15$

$a \cdot 5^2 - 5 = -15$
$25a - 5 = -15$
$25a = -10$
$a = -\frac{2}{5}$

Dus $y = -\frac{2}{5}(x-1)^2 - 5$.

b $-\frac{2}{5}(x-1)^2 - 5 = -\frac{2}{5}(x^2 - 2x + 1) - 5 = -\frac{2}{5}x^2 + \frac{4}{5}x - \frac{2}{5} - 5 = -\frac{2}{5}x^2 + \frac{4}{5}x - 5\frac{2}{5}$

Dus $y = -\frac{2}{5}x^2 + \frac{4}{5}x - 5\frac{2}{5}$.

bladzijde 157

8 De top is (4, 12), dus $y = a(x-4)^2 + 12$.
Door (0, 0), dus $a \cdot (0-4)^2 + 12 = 0$

$16a = -12$
$a = -\frac{3}{4}$

Dus $y = -\frac{3}{4}(x-4)^2 + 12$.

$-\frac{3}{4}(x-4)^2 + 12 = -\frac{3}{4}(x^2 - 8x + 16) + 12 = -\frac{3}{4}x^2 + 6x - 12 + 12 = -\frac{3}{4}x^2 + 6x$

Dus $y = -\frac{3}{4}x^2 + 6x$.

Dus $a = -\frac{3}{4}$ en $b = 6$.

9 $(-3, -6)$ invullen geeft $a \cdot (-3)^2 - 5 \cdot -3 + c = -6$, ofwel $9a + c = -21$.
$(1, -2)$ invullen geeft $a \cdot 1^2 - 5 \cdot 1 + c = -2$, ofwel $a + c = 3$.

$$\begin{cases} 9a + c = -21 \\ a + c = 3 \end{cases}$$
$$8a = -24 \quad (-)$$
$$\left.\begin{array}{l} a = -3 \\ a + c = 3 \end{array}\right\} \begin{array}{l} -3 + c = 3 \\ c = 6 \end{array}$$

Dus $y = -3x^2 - 5x + 6$.

10 $y = ax + b$
$(-1, 4)$ invullen geeft $-a + b = 4$.
$(5, -8)$ invullen geeft $5a + b = -8$.

$$\begin{cases} -a + b = 4 \\ 5a + b = -8 \end{cases}$$
$$-6a = 12 \quad (+)$$
$$\left.\begin{array}{l} a = -2 \\ -a + b = 4 \end{array}\right\} \begin{array}{l} b + 2 = 4 \\ b = 2 \end{array}$$

$y = px^2 + q$
$(-1, 4)$ invullen geeft $p + q = 4$.
$(5, -8)$ invullen geeft $25p + q = -8$.

$$\begin{cases} p + q = 4 \\ 25p + q = -8 \end{cases}$$
$$-24p = 12 \quad (-)$$
$$\left.\begin{array}{l} p = -\frac{1}{2} \\ p + q = 4 \end{array}\right\} \begin{array}{l} -\frac{1}{2} + q = 4 \\ q = 4\frac{1}{2} \end{array}$$

Dus $a = -2$, $b = 2$, $p = -\frac{1}{2}$ en $q = 4\frac{1}{2}$.

11

a $x^8 = 256$
$x = \sqrt[8]{256} = 2 \vee x = -\sqrt[8]{256} = -2$

b $x^3 = -216$
$x = \sqrt[3]{-216} = -6$

c $4x^4 + 8 = 7$
$4x^4 = -1$
$x^4 = -\frac{1}{4}$
geen oplossing

d $5 - x^5 = -4$
$-x^5 = -9$
$x^5 = 9$
$x = \sqrt[5]{9}$

e $9(x - 1)^4 = 144$
$(x - 1)^4 = 16$
$x - 1 = 2 \vee x - 1 = -2$
$x = 3 \vee x = -1$

f $\frac{1}{4}(2x - 7)^7 - 12 = -44$
$\frac{1}{4}(2x - 7)^7 = -32$
$(2x - 7)^7 = -128$
$2x - 7 = -2$
$2x = 5$
$x = 2\frac{1}{2}$

12 **a** $5x^3 - 1 = 9$
$5x^3 = 10$
$x^3 = 2$
$x = \sqrt[3]{2}$
$x \approx 1{,}260$

b $\frac{1}{2}(x-1)^4 = 12$
$(x-1)^4 = 24$
$x - 1 = \sqrt[4]{24} \vee x - 1 = -\sqrt[4]{24}$
$x = 1 + \sqrt[4]{24} \vee x = 1 - \sqrt[4]{24}$
$x \approx 3{,}213 \vee x \approx -1{,}213$

c $(1-2x)^5 - 4 = 12$
$(1-2x)^5 = 16$
$1 - 2x = \sqrt[5]{16}$
$-2x = -1 + \sqrt[5]{16}$
$x = \frac{1}{2} - \frac{1}{2}\sqrt[5]{16}$
$x \approx -0{,}371$

d $\frac{1}{3}(4-3x)^3 + 12 = 6$
$\frac{1}{3}(4-3x)^3 = -6$
$(4-3x)^3 = -18$
$4 - 3x = -\sqrt[3]{18}$
$-3x = -4 - \sqrt[3]{18}$
$x = \frac{4}{3} + \frac{1}{3}\sqrt[3]{18}$
$x \approx 2{,}207$

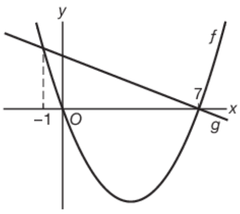
13 **a** $x^3 + 16x = 10x^2$
$x^3 - 10x^2 + 16x = 0$
$x(x^2 - 10x + 16) = 0$
$x(x-2)(x-8) = 0$
$x = 0 \vee x = 2 \vee x = 8$

b $x^4 + 56 = 15x^2$
$x^4 - 15x^2 + 56 = 0$
Stel $x^2 = u$.
$u^2 - 15u + 56 = 0$
$(u-7)(u-8) = 0$
$u = 7 \vee u = 8$
$x^2 = 7 \vee x^2 = 8$
$x = \sqrt{7} \vee x = -\sqrt{7} \vee x = \sqrt{8} \vee x = -\sqrt{8}$

c $x^4 - 7x^3 - 8x^2 = 0$
$x^2(x^2 - 7x - 8) = 0$
$x^2(x+1)(x-8) = 0$
$x = 0 \vee x = -1 \vee x = 8$

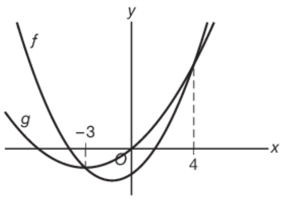
d $x^3 + 6x^2 = -9x$
$x^3 + 6x^2 + 9x = 0$
$x(x^2 + 6x + 9) = 0$
$x(x+3)(x+3) = 0$
$x = 0 \vee x = -3$

14 **a** $\underbrace{x^2 - 7x}_{f(x)} < \underbrace{-x + 7}_{g(x)}$
$x^2 - 7x = -x + 7$
$x^2 - 6x - 7 = 0$
$(x-7)(x+1) = 0$
$x = 7 \vee x = -1$



$x^2 - 7x < -x + 7$ geeft $-1 < x < 7$.

b $\underbrace{2x^2 + 5x - 12}_{f(x)} > \underbrace{x^2 + 6x}_{g(x)}$
$2x^2 + 5x - 12 = x^2 + 6x$
$x^2 - x - 12 = 0$
$(x-4)(x+3) = 0$
$x = 4 \vee x = -3$

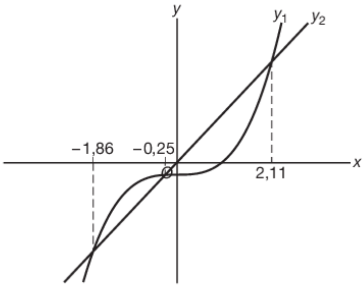


$2x^2 + 5x - 12 > x^2 + 6x$ geeft $x < -3 \vee x > 4$.

15 **a** Voer in $y_1 = 0{,}1x^3 - 2x + 2$.
De optie zero (TI) of ROOT (Casio) geeft
$x \approx -4{,}91 \vee x \approx 1{,}06 \vee x \approx 3{,}85$.
b Voer in $y_1 = 6 - 0{,}5x^2$ en $y_2 = x^3 - 8x$.
De optie intersect geeft $x \approx -2{,}66 \vee x \approx -0{,}77 \vee x \approx 2{,}93$.

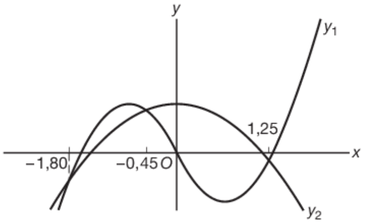
16 **a** Voer in $y_1 = -0{,}5x^3 + 4x^2 - 12$.
De optie zero (TI) of ROOT (Casio) geeft de nulpunten $-1{,}58$, 2 en 7,58.
b Voer in $y_1 = 0{,}1x^4 - 0{,}2x^3 - 4x^2 + 6x + 4$.
De optie zero (TI) of ROOT (Casio) geeft de nulpunten $-6{,}06$, $-0{,}50$, 2 en 6,56.

17 **a** $x^3 - 1 > 4x$
Voer in $y_1 = x^3 - 1$ en $y_2 = 4x$.
De optie intersect geeft
$x \approx -1{,}86 \vee x \approx -0{,}25 \vee x \approx 2{,}11$.



$x^3 - 1 > 4x$ geeft
$-1{,}86 < x < -0{,}25 \vee x > 2{,}11$.

b $x^3 - 2x < -x^2 + 1$
Voer in $y_1 = x^3 - 2x$ en $y_2 = -x^2 + 1$.
De optie intersect geeft
$x \approx -1{,}80 \vee x \approx -0{,}45 \vee x \approx 1{,}25$.



$x^3 - 2x < -x^2 + 1$ geeft
$x < -1{,}80 \vee -0{,}45 < x < 1{,}25$.

18 $h > 40$ geeft $-4t^2 + 28t + 2 > 40$.
Voer in $y_1 = -4x^2 + 28x + 2$ en $y_2 = 40$.
De optie intersect geeft $x \approx 1{,}84 \vee x \approx 5{,}16$.
De pijl is $5{,}16 - 1{,}84 \approx 3{,}3$ seconde hoger dan 40 m.